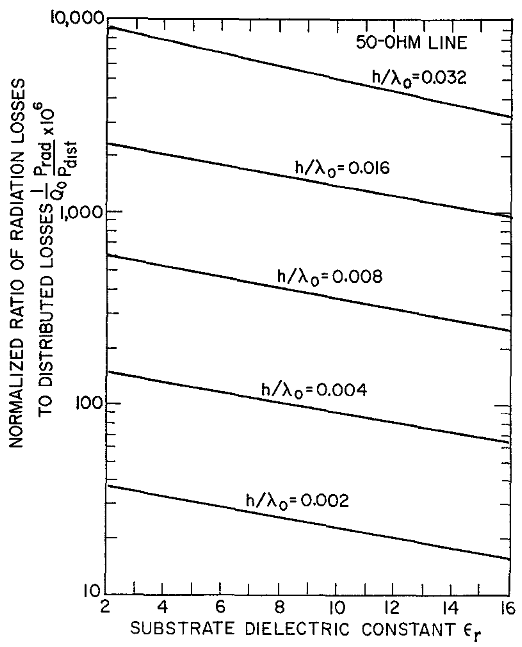


Fig. 4. Ratio of radiation loss to distributed for 50-Ω line as a function of substrate dielectric constant and frequency

Fig. 4 shows $P_{\text{rad}}/P_{\text{dist}}$ as a function of the substrate dielectric constant $\epsilon_r$ and frequency. The results shown are for a 50-Ω line, and are in good agreement with the data presented by Denlinger [8] for radiation losses in excess of 10 percent of the total loss.

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## Broad-Band Properties of a Class of TEM-Mode Hybrids

***Abstract*—The analysis of a class of *N*-port TEM-mode hybrids, operating as equal or unequal power dividers or summers, has been extended to include the use of tapered transmission lines. The analysis indicates design limitations on the VSWR and isolation characteristics, and can be applied for arbitrary frequency bandwidth and/or power division ratios. Also included are design graphs and tables that cover some common ranges of power division/summation ratios.**

Manuscript received February 19, 1971; revised May 24, 1971. This work was supported in part by the National Research Council of Canada under Grant NRC A-3725.

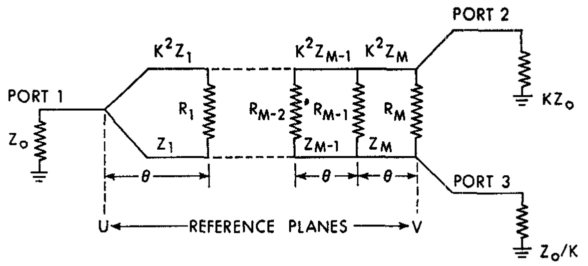



Fig. 1. Circuit for the general case of a three-port hybrid with multiple transformer sections.

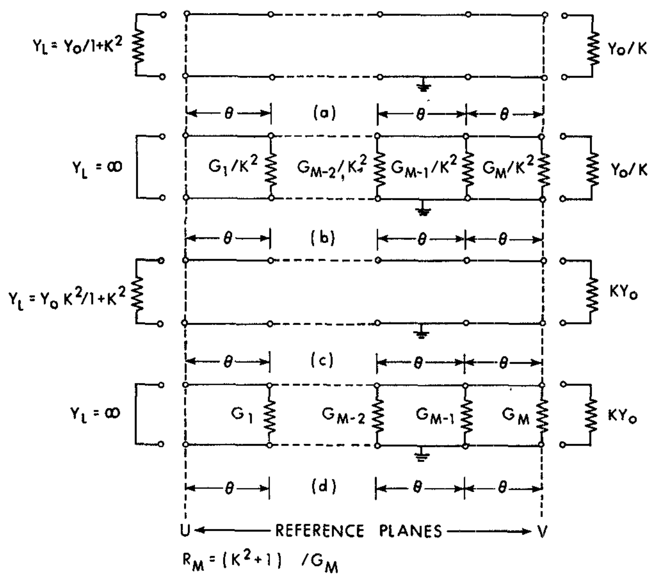



Fig. 2. Even and odd mode equivalent circuits. (a) and (b) Port 1 to port 2 in the even mode and in the odd mode, respectively. (c) and (d) Port 1 to port 3 in the even mode and in the odd mode, respectively.

### Introduction

The hybrids to be discussed can be designed for arbitrary power division ratios while maintaining equal phase in the divided parts of the circuit. By the reciprocal properties of this hybrid, signals of equal phase, having given power ratios can also be summed without loss. The summation ports are well isolated from each other within the design frequency band. The work reported here extends that reported previously [1]–[5] to include the use of tapered transmission lines and of distributed resistances to provide the needed impedance matching and isolation. Goodman [6] has given experimental evidence which indicates that it is possible and desirable to use tapered transmission lines in these hybrids. The work discussed here confirms his findings and indicates as well practical values of the distributed resistances to be used.

### General Analysis

For analysis purposes, the $N$-port hybrid can be reduced to a four-port equivalent circuit similar to that used for treating the three-port hybrid, as shown in the Appendix. This permits the properties of the hybrid to be readily obtained using the concept of even and odd modes [5].

The general circuit for the three-port hybrid discussed is shown in Figs. 1 and 2 [2], [3]. The basic circuit consists of $M$ sections with an equal number of shunt resistors. The characteristic impedances of the matching sections in each arm serve as impedance transformers to match the summation ports (2 and 3) to the divider port (1). Let the electrical length ($\theta$) be very small, and assume that the characteristic impedance of each increment of length is a stepwise approximation of the tapered transmission line. The shunt resistances can be thought of as the internal termination of the series port, of a conventional four-port hybrid network, while the factor $K$ (Fig. 1) takes into consideration unequal power division or summation ratios [2]. The summation port terminations of $KZ_0$ and $Z_0/K$ facilitate the possible further matching of these impedances to $Z_0$.

The entire circuit can be analyzed by calculating the even and the odd mode reflection coefficients ($\Gamma_e$ and $\Gamma_o$, respectively) at the reference plane $V$ in Fig. 2 [5]. By circuit symmetry, $\Gamma_e$ and $\Gamma_o$ will be the same for port 2 and for port 3. Thus, only one set of the even and odd mode equivalent circuits needs to be solved. At port 1, only the even mode exists and thus

$$\rho_1 = |\Gamma_1| = |\Gamma_e| \qquad \text{VSWR}(1) = (1+\rho_1)/(1-\rho_1). \tag{1}$$

At ports 2 and 3 both the even and the odd modes exist, so

$$\Gamma_2 = (\Gamma_e + K^2\Gamma_o)[1/(1+K^2)] \tag{2}$$

$$\Gamma_3 = (K^2\Gamma_e + \Gamma_o)[1/(1+K^2)] \tag{3}$$

$$I_{23} = I_{32} = 20 \log_{10} [|\Gamma_e - \Gamma_o| K/(1+K^2)] \tag{4}$$

where $I_{23}$ is the isolation (in dB) of port 2 with respect to port 3.

The ratio of impedances that has to be matched by the intermediate transformer sections varies with power division ratio $K^2$ as $(1+K^2)/K$. For $K=1$, a two-section Chebyshev transformer will provide a VSWR<1.11 over an octave bandwidth but, as $K$ increases, either the usable bandwidth decreases or the acceptable VSWR must be increased. A tapered transmission line of about the same length as that of two quarter-wave sections, on the other hand, provides a few octaves of bandwidth for this maximum VSWR.

From (2), (3), and (4) it is clear that an ideal design should have

$$\Gamma_e = \Gamma_o. \tag{5}$$

Then

$$\Gamma_1 = \Gamma_2 = \Gamma_3 = \Gamma_e \qquad I_{23} = I_{32} = \infty. \tag{6}$$

This ideal situation is not attainable in these hybrids due to the short-circuit condition existing for the odd mode. Alternately, one can design the hybrid such that the odd mode is minimized, leaving

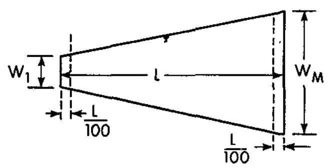
$$|\Gamma_2| < |\Gamma_e| \qquad |\Gamma_3| < |\Gamma_e| \tag{7}$$

and the isolation should then be high also, as the impedance transformers normally used are designed such that $|\Gamma_e|$ is small. The equivalent circuit for the odd mode is similar to that of a short-circuited resonator constructed from a lossy transmission line. Therefore, by appropriate choice of shunt conductances, the odd mode can be minimized.

## Hybrid Design Using Tapered Transformers and a Conductive Sheet

The even mode reflection coefficient for a tapered transmission line is readily calculated [7]. The odd mode calculations were carried out in an incremental manner by using the standard transmission line equations, commencing at the shorted end of the taper. At each step a value of shunt conductance was added, the total variation of conductance with distance along the taper being arrived at by joining the individual steps. Step size was selected so that the variation of impedance along the taper was represented with sufficient accuracy. The use of a step size of 1/100th of the total length of the taper reproduced the VSWR characteristics of the Chebyshev taper (lower cutoff frequency of 1.0 GHz, $\rho_{max}=0.05$) with an error of less than 1 percent from exact values. The calculations were carried out in an iterative manner with the computed results for previously selected conductance variations being used to select a new variation of conductance. In this manner, the difference between the computed characteristics and those when the odd mode is cancelled were rapidly minimized. This procedure was verified both experimentally and theoretically [8].

Since electromagnetic coupling between the branches of the hybrid serves mainly to effectively shorten the electrical length of the taper and rise the VSWR at all ports, the work reported here assumes there is no such coupling. Further, the results reported here assume a purely resistive conductive sheet. The introduction of some appropriate value of inductance and capacitance [8] with each incremental of conductance resulted in VSWR characteristics which were somewhat increased, uniformly across the design frequency band, and had little effect upon isolation.




$R_1 = \frac{W_1}{L/100} R_S \quad W_1 = \frac{R_1}{R_S}\frac{L}{100} \quad W_M = \frac{R_M}{R_S}\frac{L}{100}$

Fig. 3. Geometric shape of thin-film resistive sheet for use between arms 2 and 3.

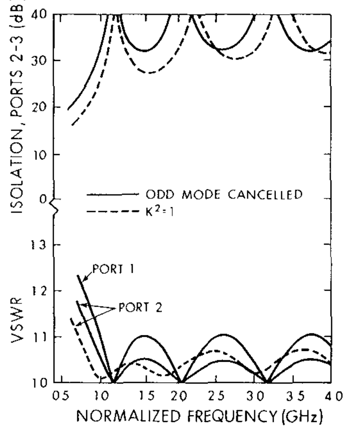



Fig. 4. Computed characteristics of a three-port hybrid using a Chebyshev tapered transformer in arms 2 and 3 (all ports assumed match terminated).

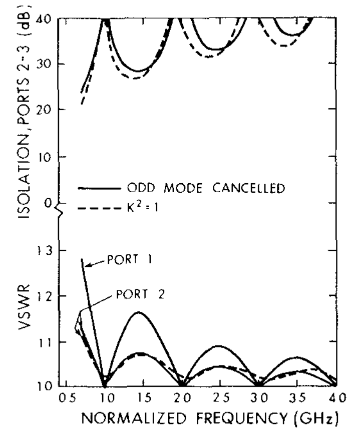



Fig. 5. Computed characteristics of a three-port hybrid using an exponential tapered transformer in arms 2 and 3 (all ports assumed match terminated).

It was noted during the computations that the variation of conductance with distance which produced best results was nearly linear. Further, the resistance distributions could be varied by ±10 percent with little effect upon the resulting VSWR and isolation. Hence, the values of resistance distributions given in Table I are linear, which has a significant practical advantage. The geometric form of the resistance sheet is shown in Fig. 3, and the values of $R_1$ and $R_m$ for various values of $K^2$ are given in Table I; the resistance values are normalized to $Z_0$ of Fig. 1. Using these linear variations of resistance, the VSWR and isolation characteristics for the various ports are shown in Fig. 4 for a Chebyshev taper and in Fig. 5 for an exponential taper. For purposes of clarity the frequency scale of

TABLE I
RESISTANCE VALUES FOR THREE-PORT HYBRIDS USING TAPERED IMPEDANCE TRANSFORMER SYSTEMS

| Power division ratio, $K^2$ | | 1 | 2 | 3 |
|---|---|---|---|---|
| Chebyshev taper | $R_1=$ | 2.000 | 6.000 | 8.000 |
| | $R_M=$ | 200.000 | 228.750 | 252.193 |
| Exponential taper | $R_1=$ | 12.000 | 14.000 | 16.000 |
| | $R_M=$ | 212.000 | 234.750 | 260.193 |

these figures is limited but the results shown are typical for higher frequencies as well. In Fig. 4 the frequency scale is normalized to the lower cutoff frequency in gigahertz when the maximum reflection coefficient is 0.05. The characteristics for $K^2>1$ are very similar to those shown in Fig. 4 with the isolation showing about a 2–3-dB improvement and the VSWR's for ports 2 and 3 being less than 1.09. In Fig. 5, the frequency scale is normalized to $\beta l=\pi$, where the electrical length ($l$) of the taper is 15 cm. Again, the characteristics for $K^2>1$ are similar to those plotted in Fig. 5. The magnitude of the peak VSWR (above cutoff) for port 1 increases as $K^2$ increases due to the fixed length of $l$ but remains less than 1.2. VSWRs for ports 2 and 3 have peak values less than 1.15 and the isolation shows an improvement of about 2 dB.

## CONCLUSION

The work reported here has extended the analysis of the $N$-port hybrid to include the use of tapered transmission lines. It has also been shown that the required isolation can be provided by a linear distribution of resistances along the length of the taper. A theoretical limit on the VSWR and isolation characteristics has been presented, along with designs that closely approach this limit. Generally, it has been found that the VSWR and isolation characteristics of the unequal power divider/summer are closely similar to those of the equal power divider/summer.

## APPENDIX

### ANALYSIS OF THE $N$-PORT HYBRID FOR ARBITRARY POWER DIVISION

When the $N$-port hybrid is used for uneven power division, the fractional power level in each arm is determined by the relative admittance of each arm. Therefore, in the equivalent circuit, the divider port characteristic admittance is split into separate admittances such that, at the reference plane of the junction, (Fig. 1, $U$) we find the following.

1) The ratio of the admittance of that branch to the characteristic admittance of the divider port is equal to the fractional power level in that branch; i.e.,

$$\frac{P_2}{P_1}=\frac{Y_{i2}}{Y_{01}}=K_2$$
$$\frac{P_3}{P_1}=\frac{Y_{i3}}{Y_{01}}=K_3\cdots\frac{P_N}{P_1}=\frac{Y_{iN}}{Y_{01}}=K_N. \tag{8}$$

2) The parallel combination of the input admittance of the branches is equal to the divider port characteristic admittance:

$$Y_{01}=Y_{i2}+Y_{i3}+\cdots+Y_{iN}. \tag{9}$$

Since the sum of the fractional divided powers equals unity when the matched condition (9) exists,

$$\sum_{j=2}^{N}K_j=1. \tag{10}$$

Now the analysis of Parad and Moynihan [2] can be adapted to the $N$-branch case if we consider that the effect on branch 2 of all the other branches: 3, 4, $\cdots$, $N$, combined, has to be equivalent to the effect of the third branch on the second branch for the three-branch hybrid. From this reasoning, it readily follows that the output impedances $R_2, R_3, \cdots, R_N$ should be chosen to be

$$R_2=Z_{01}\sqrt{(K_3+K_4+\cdots+K_N)/K_2}$$
$$R_3=Z_{01}\sqrt{(K_2+K_4+K_5+\cdots+K_N)/K_3}$$
$$\cdots\cdots\cdots$$
$$R_N=Z_{01}\sqrt{(K_2+K_3+\cdots K_{N-1})/K_N} \tag{11}$$

so that the characteristic admittances of the quarter-wave transformers in arms 2, 3, $\cdots$, $N$ are

$$Y_{02}=\sqrt{Y_{i2}G_2}=Y_{01}\sqrt{K_2}[K_2/(K_3+K_4+\cdots+K_N)]^{1/2}$$
$$Y_{03}=\sqrt{Y_{i3}G_3}=Y_{01}\sqrt{K_3}[K_3/(K_2+K_4+\cdots+K_N)]^{1/2}$$
$$\cdots\cdots\cdots$$
$$Y_{0N}=\sqrt{Y_{iN}G_N}=Y_{01}\sqrt{K_N}[K_N/(K_2+K_3+\cdots+K_{N-1})]^{1/2}. \tag{12}$$

The isolation resistors required are thus

$$R_{i2}=Z_{01}\sqrt{(K_3+K_4+\cdots+K_N)/K_2}$$
$$R_{i3}=Z_{01}\sqrt{(K_2+K_4+\cdots+K_N)/K_3}$$
$$\cdots\cdots\cdots$$
$$R_{iN}=Z_{01}\sqrt{(K_2+K_3+\cdots+K_{N-1})/K_N} \tag{13}$$

for the special case of a three-port hybrid, we have

$$K^2=K_3/K_2. \tag{14}$$

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# A Wide-Band Nearly Constant Susceptance Waveguide Element

***Abstract*—Experimental results are presented for a movable metal iris which exhibits a nearly frequency-independent susceptance. This characteristic is related to the susceptance of a centered capacitive obstacle in a waveguide modified by an empirical frequency-dependent correction factor.**

Work with waveguide cavities for solid-state microwave devices has led to a movable iris characterized by a shunt susceptance that is nearly constant with frequency. The iris is constructed of a thin rectangular metal strip mounted on a low-loss foam plastic block, as shown in Fig. 1. The block has the same dimensions as the interior of the waveguide and is made long enough to prevent the metal strip from becoming skewed with respect to the waveguide walls. The shim is centered on the foam block, so there is no metal-to-metal contact.

Manuscript received March 3, 1971, revised June 7, 1971.